

# Study of $f_0(980)$ and $f_0(1500)$ from $B_s \rightarrow f_0(980)\pi, f_0(1500)\pi$ Decays

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## Abstract

In this paper, we analyze the scalar mesons  $f_0(980)$  and  $f_0(1500)$  from the decays  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0, f_0(1500)\pi^0$  within Perturbative QCD approach. From the leading order calculations, we find that (a) in the allowed mixing angle ranges, the branching ratio of  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0$  is about  $(1.0 \sim 1.6) \times 10^{-7}$ , which is smaller than that of  $\bar{B}_s^0 \rightarrow f_0(980)K^0$  (the difference is a few times even one order); (b) the decay  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$  is better to distinguish between the lowest lying state or the first excited state for  $f_0(1500)$ , because the branching ratios for two scenarios have about one-order difference in most of the mixing angle ranges; and (c) the direct CP asymmetries of  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$  for two scenarios also exists great difference. In scenario II, the variation range of the value  $\mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0)$  according to the mixing angle is very small, except for the values corresponding to the mixing angles being near  $90^\circ$  or  $270^\circ$ , while the variation range of  $\mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0)$  in scenario I is very large. Compared with the future data for the decay  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$ , it is ease to determine the nature of the scalar meson  $f_0(1500)$ .

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## I. INTRODUCTION

For the underlying structure of the scalar mesons is still under controversy, there are two typical schemes for the classification to them [1, 2]. The nonet mesons below 1 GeV, including  $f_0(600)$ ,  $f_0(980)$ ,  $K^*(800)$  and  $a_0(980)$ , are usually viewed as the lowest lying  $q\bar{q}$  states, while the nonet ones near 1.5 GeV, including  $f_0(1370)$ ,  $f_0(1500)/f_0(1700)$ ,  $K^*(1430)$  and  $a_0(1450)$ , are suggested as the first excited states. Here we denote this scheme as scenario I, and the following scheme as scenario II: the nonet mesons near 1.5 GeV are treated as  $q\bar{q}$  ground states, while the nonet mesons below 1 GeV are exotic states beyond the quark model such as four-quark bound states. In order to uncover the inner structures, many approaches are used to research the modes of  $B_{u,d}$  decaying into a scalar and a pseudoscalar (vector) meson, such as the generalized factorization approach [3], QCD factorization approach (QCDF) [4–6], Perturbative QCD (PQCD) approach [7–12]. On the experimental side, along with the running of the Large Hadron Collider beauty experiments (LHC-b), some of  $B_s^0$  decays involved a scalar in the final states might be observed in the Large Hadron Collider beauty experiments (LHC-b) [13, 14]. In order to make precision studies of rare decays in the B-meson systems, the LHC-b detector is designed to exploit the large number of b hadrons produced. Furthermore, it can reconstruct a B-decay vertex with very good resolution, which is essential for studying the rapidly oscillating  $B_s$  mesons. Some of  $B_s^0$  decays involved a scalar in the final states can also serve as an ideal platform to probe the natures of these scalar mesons. So the studies of these decay modes for  $B_s^0$  are necessary in the next a few years.

In this paper, we will study the branching ratios and the direct CP asymmetries of  $\bar{B}_s^0 \rightarrow f_0(980)\pi$ ,  $f_0(1500)\pi$  within Perturbative QCD approach based on  $k_T$  factorization. It is organized as follows: In Sect.II, we introduce the input parameters including the decay constants and light-cone distribution amplitudes. In Sec.III, we then apply PQCD approach to calculate analytically the branching ratios and CP asymmetries for our considered decays. The final part contains our numerical results and discussions.

## II. INPUT PARAMETERS

In order to make quantitative predictions, we identify  $f_0(980)$  as a mixture of  $s\bar{s}$  and  $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ , that is

$$|f_0(980)\rangle = |s\bar{s}\rangle \cos \theta + |n\bar{n}\rangle \sin \theta, \quad (1)$$

where the mixing angle  $\theta$  is taken in the ranges of  $25^\circ < \theta < 40^\circ$  and  $140^\circ < \theta < 165^\circ$  [15]. Certainly,  $f_0(1500)$  can be treated as a  $q\bar{q}$  state in both scenario I and II. We consider that the meson  $f_0(1500)$  and  $f_0(980)$  have the same component structure but with different mixing angle.

For the neutral scalar meson  $f_0(980)$ ,  $f_0(1500)$  cannot be produced via the vector current, we have  $\langle f_0(p) | \bar{q}_2 \gamma_\mu q_1 | 0 \rangle = 0$ . Taking the mixing into account, the scalar current  $\langle f_0(p) | \bar{q}_2 q_1 | 0 \rangle = m_S f_S$  can be written as:

$$\langle f_0^n | d\bar{d} | 0 \rangle = \langle f_0^n | u\bar{u} | 0 \rangle = \frac{1}{\sqrt{2}} m_{f_0} \tilde{f}_{f_0}^n, \quad \langle f_0^s | s\bar{s} | 0 \rangle = m_{f_0} \tilde{f}_{f_0}^s, \quad (2)$$

where  $f_0^{(n,s)}$  represent for the quark flavor states for  $n\bar{n}$  and  $s\bar{s}$  components of  $f_0$  meson, respectively. For the scalar decay constants  $\tilde{f}_{f_0}^n$  and  $\tilde{f}_{f_0}^s$  are very close[5], we can assume  $\tilde{f}_{f_0}^n = \tilde{f}_{f_0}^s$  and denote them as  $\bar{f}_{f_0}$  in the following.

The twist-2 and twist-3 light-cone distribution amplitudes (LCDAs) for different components of  $f_0$  are defined by:

$$\langle f_0(p) | \bar{q}(z)_i q(0)_j | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} \{ p \Phi_{f_0}(x) + m_{f_0} \Phi_{f_0}^S(x) + m_{f_0} (\not{n}_+ \not{n}_- - 1) \Phi_{f_0}^T(x) \}_{jl}, \quad (3)$$

where we assume  $f_0^n(p)$  and  $f_0^s(p)$  are same and denote them as  $f_0(p)$ ,  $n_+$  and  $n_-$  are light-like vectors:  $n_+ = (1, 0, 0_T)$ ,  $n_- = (0, 1, 0_T)$ . The normalization can be related to the decay constants:

$$\int_0^1 dx \Phi_{f_0}(x) = \int_0^1 dx \Phi_{f_0}^T(x) = 0, \quad \int_0^1 dx \Phi_{f_0}^S(x) = \frac{\bar{f}_{f_0}}{2\sqrt{2N_c}}. \quad (4)$$

The wave function for  $\pi$  meson is given as [16]

$$\Phi_\pi(P, x, \zeta) \equiv \frac{1}{\sqrt{2N_c}} \gamma_5 [\not{P} \Phi_\pi^A(x) + m_0^\pi \Phi_\pi^P(x) + \zeta m_0^\pi (\not{p} \not{n} - v \cdot n) \Phi_\pi^T(x)]. \quad (5)$$

where  $P$  and  $x$  are the momentum and the momentum fraction of  $\pi$  meson, respectively. The parameter  $\zeta$  is either +1 or -1 depending on the assignment of the momentum fraction  $x$ .

In general, the  $B_s$  meson is treated as heavy-light system and its Lorentz structure can be written as[17, 18]

$$\Phi_{B_s} = \frac{1}{\sqrt{2N_c}} (\not{P}_{B_s} + M_{B_s}) \gamma_5 \phi_{B_s}(k_1). \quad (6)$$

For the contribution of  $\bar{\phi}_{B_s}$  is numerically small [19] and has been neglected.

### III. THEORETICAL FRAMEWORK AND PERTURBATIVE CALCULATIONS

Under the two-quark model for the scalar mesons supposition, we would like to use PQCD approach to study  $\bar{B}_s^0 \rightarrow f_0(980)\pi, f_0(1500)\pi$  decays. In this approach, the decay amplitude is separated into soft, hard, and harder dynamics characterized by different energy scales  $(t, m_{B_s}, M_W)$ . It is conceptually written as the convolution,

$$\mathcal{A}(\bar{B}_s^0 \rightarrow f_0\pi) \sim \int d^4k_1 d^4k_2 d^4k_3 \text{Tr} [C(t) \Phi_{B_s}(k_1) \Phi_{f_0}(k_2) \Phi_\pi(k_3) H(k_1, k_2, k_3, t)], \quad (7)$$

where  $k_i$ 's are momenta of anti-quarks included in each mesons, and Tr denotes the trace over Dirac and color indices.  $C(t)$  is the Wilson coefficient which results from the radiative corrections at short distance. The function  $H(k_1, k_2, k_3, t)$  describes the four quark operator and the spectator quark connected by a hard gluon whose  $q^2$  is in the

order of  $\bar{\Lambda}M_{B_s}$ , and includes the  $\mathcal{O}(\sqrt{\Lambda M_{B_s}})$  hard dynamics. Therefore, this hard part  $H$  can be perturbatively calculated.

Since the  $b$  quark is rather heavy, we consider the  $\bar{B}_s^0$  meson at rest for simplicity. It is convenient to use light-cone coordinate  $(p^+, p^-, \mathbf{p}_T)$  to describe the meson's momenta,

$$p^\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^3), \quad \text{and} \quad \mathbf{p}_T = (p^1, p^2). \quad (8)$$

Using these coordinates the  $\bar{B}_s^0$  meson and the two final state meson momenta can be written as

$$P_{B_s} = \frac{M_{B_s}}{\sqrt{2}}(1, 1, \mathbf{0}_T), \quad P_2 = \frac{M_{B_s}}{\sqrt{2}}(1, 0, \mathbf{0}_T), \quad P_3 = \frac{M_{B_s}}{\sqrt{2}}(0, 1, \mathbf{0}_T), \quad (9)$$

respectively. The meson masses have been neglected. Putting the anti-quark momenta in  $\bar{B}_s^0$ ,  $f_0$  and  $\pi^0$  mesons as  $k_1$ ,  $k_2$ , and  $k_3$ , respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}), \quad k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T}), \quad k_3 = (0, x_3 P_3^-, \mathbf{k}_{3T}). \quad (10)$$

For our considered decay channels, the integration over  $k_1^-$ ,  $k_2^-$ , and  $k_3^+$  in eq.(7) will lead to

$$\mathcal{A}(\bar{B}_s^0 \rightarrow f_0 \pi^0) \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \cdot \text{Tr} [C(t) \Phi_{B_s}(x_1, b_1) \Phi_{f_0}(x_2, b_2) \Phi_\pi(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)}], \quad (11)$$

where  $b_i$  is the conjugate space coordinate of  $k_{iT}$ , and  $t$  is the largest energy scale in function  $H(x_i, b_i, t)$ . The large logarithms ( $\ln m_W/t$ ) coming from QCD radiative corrections to four-quark operators are included in the Wilson coefficients  $C(t)$ . The large double logarithms ( $\ln^2 x_i$ ) on the longitudinal direction are summed by the threshold resummation [20], and they lead to  $S_t(x_i)$ , which smears the end-point singularities on  $x_i$ . The last term,  $e^{-S(t)}$ , is the Sudakov form factor, which suppresses the soft dynamics effectively [21]. Thus it makes the perturbative calculation of the hard part  $H$  applicable at intermediate scale, i.e.,  $M_{B_s}$  scale.

We will calculate analytically the function  $H(x_i, b_i, t)$  for  $\bar{B}_s^0 \rightarrow f_0 \pi^0$  decays in the leading-order and give the convoluted amplitudes. For our considered decays, the related weak effective Hamiltonian  $\mathcal{H}_{eff}$  can be written as [22]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qs}^* \left[ (C_1(\mu) O_1^q(\mu) + C_2(\mu) O_2^q(\mu)) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right], \quad (12)$$

with the Fermi constant  $G_F = 1.16639 \times 10^{-5} \text{GeV}^{-2}$ , and the CKM matrix elements  $V$ . We specify below the operators in  $\mathcal{H}_{eff}$  for  $b \rightarrow s$  transition:

$$\begin{aligned} O_1^u &= \bar{s}_\alpha \gamma^\mu L u_\beta \cdot \bar{u}_\beta \gamma_\mu L b_\alpha, & O_2^u &= \bar{s}_\alpha \gamma^\mu L u_\alpha \cdot \bar{u}_\beta \gamma_\mu L b_\beta, \\ O_3 &= \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu L q'_\beta, & O_4 &= \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu L q'_\alpha, \\ O_5 &= \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu R q'_\beta, & O_6 &= \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu R q'_\alpha, \\ O_7 &= \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot \sum_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu R q'_\beta, & O_8 &= \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu R q'_\alpha, \\ O_9 &= \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot \sum_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu L q'_\beta, & O_{10} &= \frac{3}{2} \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu L q'_\alpha, \end{aligned} \quad (13)$$

where  $\alpha$  and  $\beta$  are the  $SU(3)$  color indices;  $L$  and  $R$  are the left- and right-handed projection operators with  $L = (1 - \gamma_5)$ ,  $R = (1 + \gamma_5)$ . The sum over  $q'$  runs over the quark fields that are active at the scale  $\mu = O(m_{B_s})$ , i.e., ( $q' \in \{u, d, s, c, b\}$ ).

We will show the whole amplitude for each diagram including wave functions. There are 8 type diagrams contributing to the  $\bar{B}_s^0 \rightarrow f_0 \pi^0$  decays are illustrated in Fig.1. We first calculate the usual factorizable diagrams (a) and (b). Operators  $O_{1,2,3,4,9,10}$  are  $(V - A)(V - A)$  currents, and the operators  $O_{5,6,7,8}$  have the structure of  $(V - A)(V + A)$ , the sum of the their amplitudes are written as  $F_{ef_0}$  and  $F_{ef_0}^{P1}$ , respectively.

$$\begin{aligned} F_{ef_0} &= F_{ef_0}^{P1} = 8\pi C_F m_{B_s}^4 f_\pi \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_{B_s}(x_1, b_1) \\ &\times \left\{ [(1 + x_2)\Phi_{f_0}(x_2) - r_{f_0}(1 - 2x_2)(\Phi_{f_0}^S(x_2) + \Phi_{f_0}^T(x_2))] \right. \\ &\times E_{ei}(t)h_e(x_1, x_2, b_1, b_2) - 2r_{f_0}\Phi_{f_0}^S(x_2)E_{ei}(t')h_e(x_2, x_1, b_2, b_1) \left. \right\}, \end{aligned} \quad (14)$$

where  $f_\pi$  is the decay constant of  $\pi$  meson,  $r_{f_0} = m_{f_0}/m_{B_s}$ .

In some other cases, we need to do Fierz transformation for the corresponding operators to get right flavor and color structure for factorization to work. We may get  $(S - P)(S + P)$  operators from  $(V - A)(V + A)$  ones. For these  $(S - P)(S + P)$  operators, Fig. 1(a) and 1(b) give

$$\begin{aligned} F_{ef_0}^{P2} &= 16\pi C_F m_{B_s}^4 f_\pi r_\pi \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_{B_s}(x_1, b_1) \\ &\times \left\{ -[\Phi_{f_0}(x_2) + r_{f_0}(x_2\Phi_{f_0}^T(x_2) - (x_2 + 2)\Phi_{f_0}^S(x_2))] \right. \\ &\times E_{ei}(t)h_e(x_1, x_2, b_1, b_2) + 2r_{f_0}\Phi_{f_0}^S(x_2)E_{ei}(t')h_e(x_2, x_1, b_2, b_1) \left. \right\}, \end{aligned} \quad (15)$$

where  $r_\pi = m_0^\pi/m_{B_s}$ .

For the non-factorizable diagrams 1(c) and 1(d), all three meson wave functions are involved. The integration of  $b_2$  can be performed using  $\delta$  function  $\delta(b_3 - b_2)$ , leaving only integration of  $b_1$  and  $b_3$ . Here we have two kinds of contributions:  $M_{ef_0}$ ,  $M_{ef_0}^{P1}$  and  $M_{ef_0}^{P2}$  describe the contributions from the  $(V - A)(V - A)$ ,  $(V - A)(V + A)$  and  $(S - P)(S + P)$  operators, respectively.

$$\begin{aligned} \mathcal{M}_{ef_0} &= 32\pi C_F m_{B_s}^4 / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \Phi_{B_s}(x_1, b_1) \Phi_\pi^A(x_3) \\ &\times \left\{ -[(x_3 - 1)\Phi_{f_0}(x_2) - r_{f_0}x_2(\Phi_{f_0}^S(x_2) - \Phi_{f_0}^T(x_2))]E'_{ei}(t)h_n(x_1, 1 - x_3, x_2, b_1, b_3) \right. \\ &\left. - [(x_2 + x_3)\Phi_{f_0}(x_2) + r_{f_0}x_2(\Phi_{f_0}^S(x_2) + \Phi_{f_0}^T(x_2))]E'_{ei}(t')h_n(x_1, x_3, x_2, b_1, b_3) \right\} \quad (16) \end{aligned}$$

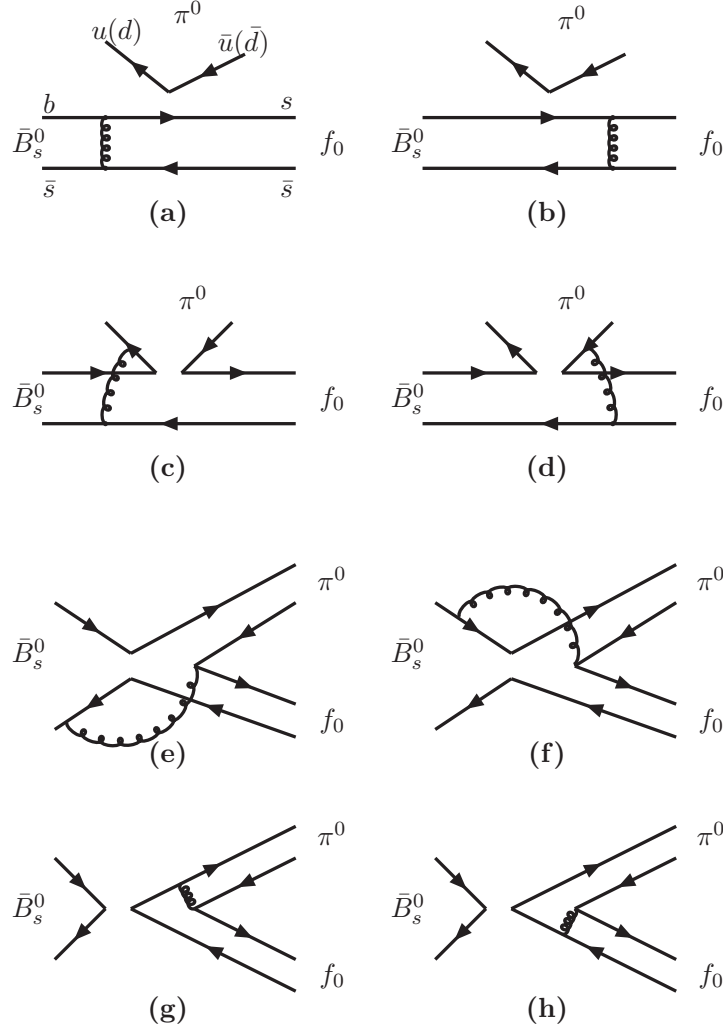


FIG. 1: Diagrams contributing to the  $\bar{B}_s^0 \rightarrow f_0 \pi^0$  decays.

$$\begin{aligned}
\mathcal{M}_{ef_0}^{P1} = & 32\pi C_F m_{B_s}^4 / \sqrt{2N_C r_\pi} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \\
& \times \Phi_{B_s}(x_1, b_1) \left\{ [(x_3 - 1)\Phi_{f_0}(x_2)(\Phi_\pi^P(x_3) + \Phi_\pi^T(x_3)) \right. \\
& + r_{f_0}\Phi_{f_0}^T(x_2)((x_2 + x_3 - 1)\Phi_\pi^P(x_3) + (-x_2 + x_3 - 1)\Phi_\pi^T(x_3)) \\
& + r_{f_0}\Phi_{f_0}^S(x_2)((x_2 - x_3 + 1)\Phi_\pi^P(x_3) - (x_2 + x_3 - 1)\Phi_\pi^T(x_3))] \\
& \times E'_{ei}(t)h_n(x_1, 1 - x_3, x_2, b_1, b_3) + E'_{ei}(t')h_n(x_1, x_3, x_2, b_1, b_3) \\
& \times [-x_3\Phi_{f_0}(x_2)(\Phi_\pi^T(x_3) - \Phi_\pi^P(x_3)) - r_{f_0}x_3(\Phi_{f_0}^S(x_2) - \Phi_{f_0}^T(x_2)) \\
& \times (\Phi_\pi^P(x_3) - \Phi_\pi^T(x_3)) - r_{f_0}x_2(\Phi_{f_0}^S(x_2) + \Phi_{f_0}^T(x_2))(\Phi_\pi^P(x_3) + \Phi_\pi^T(x_3))] \left. \right\}, \quad (17)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{ef_0}^{P2} = & -32\pi C_F m_{B_s}^4 / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \Phi_{B_s}(x_1, b_1) \Phi_\pi^A(x_3) \\
& \times \left\{ [(x_3 - x_2 - 1) \Phi_{f_0}(x_2) - r_{f_0} x_2 (\Phi_{f_0}^S(x_2) + \Phi_{f_0}^T(x_2))] \right. \\
& \times E'_{ei}(t) h_n(x_1, 1 - x_3, x_2, b_1, b_3) + E'_{ei}(t') h_n(x_1, x_3, x_2, b_1, b_3) \\
& \left. \times [x_2 \Phi_{f_0}(x_2) + r_{f_0} x_2 (\Phi_{f_0}^S(x_2) - \Phi_{f_0}^T(x_2))] \right\}. \quad (18)
\end{aligned}$$

For the non-factorizable annihilation diagrams (e) and (f), again all three wave functions are involved. For the  $(V - A)(V - A)$  and  $(S - P)(S + P)$  operators, the results are

$$\begin{aligned}
\mathcal{M}_{af_0} = & -32\pi C_F m_{B_s}^4 / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \\
& \times \Phi_{B_s}(x_1, b_1) \left\{ E'_{ai}(t) h_{na}(x_1, x_3, x_2, b_1, b_3) [x_3 \Phi_\pi^A(x_3) \Phi_{f_0}(x_2) \right. \\
& + r_\pi r_{f_0} \Phi_{f_0}^T(x_2) ((x_2 - x_3 + 1) \Phi_\pi^T(x_3) - (x_2 + x_3 - 1) \Phi_\pi^P(x_3)) \\
& + r_\pi r_{f_0} \Phi_{f_0}^S(x_2) ((-x_2 + x_3 + 3) \Phi_\pi^P(x_3) + (x_2 + x_3 - 1) \Phi_\pi^T(x_3))] \\
& + E'_{ai}(t') h'_{na}(x_1, x_3, x_2, b_1, b_3) [(x_2 - 1) \Phi_\pi^A(x_3) \Phi_{f_0}(x_2) \\
& + r_\pi r_{f_0} \Phi_{f_0}^T(x_2) ((-x_2 + x_3 + 1) \Phi_\pi^T(x_3) - (x_2 + x_3 - 1) \Phi_\pi^P(x_3)) \\
& \left. + r_\pi r_{f_0} \Phi_{f_0}^S(x_2) ((x_2 - x_3 - 1) \Phi_\pi^P(x_3) + (x_2 + x_3 - 1) \Phi_\pi^T(x_3))] \right\}, \quad (19)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{af_0}^{P2} = & -32\pi C_F m_B^4 / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \Phi_{B_s}(x_1, b_1) \\
& \times \left\{ [(x_2 - 1) \Phi_{f_0}(x_2) \Phi_\pi^A(x_3) - 4r_\pi r_{f_0} \Phi_{f_0}^S(x_2) \Phi_\pi^P(x_3) + r_\pi r_{f_0} ((x_3 - x_2 - 1) \right. \\
& \times (\Phi_\pi^P(x_3) \Phi_{f_0}^S(x_2) + \Phi_\pi^T(x_3) \Phi_{f_0}^T(x_2)) - (x_2 + x_3 - 1) (\Phi_\pi^P(x_3) \Phi_{f_0}^T(x_2) \\
& - \Phi_\pi^T(x_3) \Phi_{f_0}^S(x_2))] E'_{ai}(t) h_{na}(x_1, x_3, x_2, b_1, b_3) + E'_{ai}(t') h'_{na}(x_1, x_3, x_2, b_1, b_3) \\
& \times [x_3 \Phi_{f_0}(x_2) \Phi_\pi^A(x_3) + x_3 r_\pi r_{f_0} (\Phi_{f_0}^S(x_2) - \Phi_{f_0}^T(x_2)) (\Phi_\pi^P(x_3) + \Phi_\pi^T(x_3)) \\
& \left. + r_\pi r_{f_0} (1 - x_2) (\Phi_{f_0}^S(x_2) + \Phi_{f_0}^T(x_2)) (\Phi_\pi^P(x_3) - \Phi_\pi^T(x_3))] \right\}. \quad (20)
\end{aligned}$$

The factorizable annihilation diagrams (g) and (h) involve only the  $\pi$  and  $f_0$  mesons' wave functions. There are three kinds of decay amplitudes for these two diagrams.  $F_{af_0}$  is for  $(V - A)(V - A)$  type operators,  $F_{af_0}^{P1}$  is for  $(V - A)(V + A)$  type operators, while

$F_{af_0}^{P2}$  is for  $(S - P)(S + P)$  type operators:

$$\begin{aligned}
F_{af_0} = F_{af_0}^{P1} = & 8\pi C_F m_{B_s}^4 f_{B_s} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \left\{ [(x_2 - 1)\Phi_\pi^A(x_3)\Phi_{f_0}(x_2) \right. \\
& + 2r_\pi r_{f_0}(x_2 - 2)\Phi_\pi^P(x_3)\Phi_{f_0}^S(x_2) - 2r_\pi r_{f_0} x_2 \Phi_\pi^P(x_3)\Phi_{f_0}^T(x_2)] \\
& \times E_{ai}(t)h_a(x_3, 1 - x_2, b_3, b_2) + E_{ai}(t')h_a(1 - x_2, x_3, b_2, b_3) [x_3\Phi_\pi^A(x_3)\Phi_{f_0}(x_2) \\
& \left. + 2r_\pi r_{f_0}\Phi_{f_0}^S(x_2)((x_3 + 1)\Phi_\pi^P(x_3) + (x_3 - 1)\Phi_\pi^T(x_3))] \right\}, \quad (21)
\end{aligned}$$

$$\begin{aligned}
F_{af_0}^{P2} = & 16\pi C_F m_{B_s}^4 f_{B_s} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\
& \times \left\{ [r_{f_0}(x_2 - 1)\Phi_\pi^A(x_3)(\Phi_{f_0}^S(x_2) + \Phi_{f_0}^T(x_2)) - 2r_\pi \Phi_\pi^P(x_3)\Phi_{f_0}(x_2)] \right. \\
& \times E_{ai}(t)h_a(x_3, 1 - x_2, b_2, b_3) - E_{ai}(t')h_a(1 - x_2, x_3, b_2, b_3) \\
& \left. \times [2r_{f_0}\Phi_\pi^A(x_3)\Phi_{f_0}^S(x_2) + r_\pi x_3\Phi_{f_0}(x_2)(\Phi_\pi^P(x_3) - \Phi_\pi^T(x_3))] \right\}. \quad (22)
\end{aligned}$$

If we exchange the  $\pi^0$  and  $f_0$  in Fig.1, the result will be different. In the considered decays, the meson  $f_0$  cannot lie in the emitted position, like  $\pi'$  position in Fig.1(a-d). So only the annihilation type diagrams left, just like Fig.1(e-h), can give contributions. They are listed as follows:

$$\begin{aligned}
\mathcal{M}_{a\pi} = & 32\pi C_F m_{B_s}^4 / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_{B_s}(x_1, b_1) \left\{ [-x_2\Phi_\pi^A(x_3) \right. \\
& \times \Phi_{f_0}(x_2) + r_\pi r_{f_0}\Phi_{f_0}^T(x_2)((x_2 + x_3 - 1)\Phi_\pi^P(x_3) + (-x_2 + x_3 + 1)\Phi_\pi^T(x_3)) \\
& + r_\pi r_{f_0}\Phi_{f_0}^S(x_2)((x_2 - x_3 + 3)\Phi_\pi^P(x_3) - (x_2 + x_3 - 1)\Phi_\pi^T(x_3))] \\
& \times E'_{ai}(t)h_{na}(x_1, x_2, x_3, b_1, b_2) - E'_{ai}(t')h'_{na}(x_1, x_2, x_3, b_1, b_2) [(x_3 - 1)\Phi_\pi^A(x_3) \\
& \times \Phi_{f_0}(x_2) + r_\pi r_{f_0}\Phi_{f_0}^S(x_2)((x_2 - x_3 + 1)\Phi_\pi^P(x_3) - (x_2 + x_3 - 1)\Phi_\pi^T(x_3)) \\
& \left. + r_\pi r_{f_0}\Phi_{f_0}^T(x_2)((x_2 + x_3 - 1)\Phi_\pi^P(x_3) - (1 + x_2 - x_3)\Phi_\pi^T(x_2))] \right\}, \quad (23)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{a\pi}^{P2} = & -32\pi C_F m_{B_s}^4 / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_{B_s}(x_1, b_1) \left\{ [4r_\pi r_{f_0}\Phi_{f_0}^S(x_2) \right. \\
& \times \Phi_\pi^P(x_3) + (x_3 - 1)\Phi_{f_0}(x_2)\Phi_\pi^A(x_3) + r_\pi r_{f_0}((x_2 - x_3 - 1)(\Phi_\pi^P(x_3)\Phi_{f_0}^S(x_2) \\
& - \Phi_\pi^T(x_3)\Phi_{f_0}^T(x_2)) - (x_2 + x_3 - 1)(\Phi_\pi^P(x_3)\Phi_{f_0}^T(x_2) - \Phi_\pi^T(x_3)\Phi_{f_0}^S(x_2))] \\
& \times E'_{ai}(t)h_{na}(x_1, x_2, x_3, b_1, b_2) + [x_2\Phi_{f_0}(x_2)\Phi_\pi^A(x_3) - x_2r_\pi r_{f_0}(\Phi_{f_0}^S(x_2) + \Phi_{f_0}^T(x_2)) \\
& \times (\Phi_\pi^P(x_3) - \Phi_\pi^T(x_3)) - r_\pi r_{f_0}(1 - x_3)(\Phi_{f_0}^S(x_2) - \Phi_{f_0}^T(x_2))(\Phi_\pi^P(x_3) + \Phi_\pi^T(x_3))] \\
& \left. \times E'_{ai}(t')h'_{na}(x_1, x_2, x_3, b_1, b_2) \right\}, \quad (24)
\end{aligned}$$



$$\begin{aligned}
F_{a\pi} = -F_{a\pi}^{P1} = & 8\pi C_F m_{B_s}^4 f_{B_s} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \left\{ [(x_3 - 1)\Phi_\pi^A(x_3)\Phi_{f_0}(x_2) \right. \\
& - 2r_\pi r_{f_0}(x_3 - 2)\Phi_\pi^P(x_3)\Phi_{f_0}^S(x_2) + 2r_\pi r_{f_0} x_3 \Phi_\pi^T(x_3)\Phi_{f_0}^S(x_2)] \\
& \times E_{ai}(t) h_a(x_2, 1 - x_3, b_2, b_3) + E_{ai}(t') h_a(1 - x_3, x_2, b_3, b_2) \\
& \left. \times [x_2 \Phi_\pi^A(x_3)\Phi_{f_0}(x_2) - 2r_\pi r_{f_0} \Phi_\pi^P(x_3)((x_2 + 1)\Phi_{f_0}^S(x_2) + (x_2 - 1)\Phi_{f_0}^T(x_2))] \right\}, \quad (25)
\end{aligned}$$

$$\begin{aligned}
F_{a\pi}^{P2} = & -16\pi C_F m_{B_s}^4 f_{B_s} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\
& \times \left\{ [r_\pi(x_3 - 1)\Phi_{f_0}(x_2)(\Phi_\pi^P(x_3) + \Phi_\pi^T(x_3)) + 2r_{f_0}\Phi_\pi(x_3)\Phi_{f_0}^S(x_2)] \right. \\
& \times E_{ai}(t) h_a(x_2, 1 - x_3, b_2, b_3) - E_{ai}(t') h_a(1 - x_3, x_2, b_3, b_2) \\
& \left. \times [2r_\pi \Phi_\pi^P(x_3)\Phi_{f_0}(x_2) + r_{f_0} x_2 \Phi_\pi^A(x_3)(\Phi_{f_0}^T(x_2) - \Phi_{f_0}^S(x_2))] \right\}. \quad (26)
\end{aligned}$$

In the above formulae, the function  $E$  are defined as:

$$E_{ei}(t) = \alpha_s(t) \exp[-S_B(t) - S_3(t)], \quad (27)$$

$$E'_{ei}(t) = \alpha_s(t) \exp[-S_B(t) - S_2(t) - S_3(t)]|_{b_3=b_1}, \quad (28)$$

$$E_{ai}(t) = \alpha_s(t) \exp[-S_2(t) - S_3(t)], \quad (29)$$

$$E'_{ai}(t) = \alpha_s(t) \exp[-S_B(t) - S_2(t) - S_3(t)]|_{b_3=b_2}, \quad (30)$$

where  $\alpha_s$  is the strong coupling constant,  $S$  is the Sudakov form factor. In our numerical analysis, we use the one-loop expression for the strong coupling constant; we use  $c = 0.4$  for the parameter in the jet function. The explicit form of  $h$  and  $S$  have been given in [23].

Combining the contributions from different diagrams, the total decay amplitudes for these decays can be written as:

$$\mathcal{M}(\bar{B}_s^0 \rightarrow f_0 \pi) = \mathcal{M}_{s\bar{s}} \times \cos \theta + \frac{1}{\sqrt{2}} \mathcal{M}_{n\bar{n}} \sin \theta, \quad (31)$$

where  $\theta$  is mixing angle, and

$$\mathcal{M}_{s\bar{s}} = V_{ub} V_{us}^* (F_{ef_0} a_2 + M_{ef_0} C_2) - \frac{3}{2} V_{tb} V_{ts}^* [F_{ef_0}^{P2} (a_9 - a_7) + M_{ef_0}^{P1} C_{10} + M_{ef_0}^{P2} C_8], \quad (32)$$

$$\begin{aligned}
\mathcal{M}_{n\bar{n}} = & V_{ub} V_{us}^* [(M_{a\pi} + M_{af_0}) C_2 + (F_{a\pi} + F_{af_0}) a_2] - \frac{3}{2} V_{tb} V_{ts}^* [(M_{a\pi} + M_{af_0}) C_{10} \\
& + (M_{a\pi}^{P2} + M_{af_0}^{P2}) C_8 + (F_{a\pi} + F_{af_0}) (a_9 - a_7)], \quad (33)
\end{aligned}$$

where the combinations of the Wilson coefficients are defined as usual [24, 25]:

$$a_1 = C_2 + C_1/3, \quad a_3 = C_3 + C_4/3, \quad a_5 = C_5 + C_6/3, \quad a_7 = C_7 + C_8/3, \quad a_9 = C_9 + C_{10}/3, \quad (34)$$

$$a_2 = C_1 + C_2/3, \quad a_4 = C_4 + C_3/3, \quad a_6 = C_6 + C_5/3, \quad a_8 = C_8 + C_7/3, \quad a_{10} = C_{10} + C_9/3. \quad (35)$$

#### IV. NUMERICAL RESULTS AND DISCUSSIONS

The twist-2 LCDA  $\Phi_{f_0}$  can be expanded in the Gegenbauer polynomials:

$$\Phi_{f_0}(x, \mu) = \frac{1}{2\sqrt{2N_c}} \bar{f}_{f_0}(\mu) 6x(1-x) \sum_{m=1}^{\infty} B_m(\mu) C_m^{3/2}(2x-1), \quad (36)$$

where  $B_m(\mu)$  and  $C_m^{3/2}(x)$  are the Gegenbauer moments and Gegenbauer polynomials, respectively. The values for Gegenbauer moments and the decay constants are taken (at scale  $\mu = 1\text{GeV}$ ) as:

$$\begin{aligned} \text{Scenario I : } \bar{f}_{f_0(980)} &= (0.37 \pm 0.02)\text{GeV}, & \bar{f}_{f_0(1500)} &= -(0.255 \pm 0.03)\text{GeV}, \\ B_1(980) &= -0.78 \pm 0.08, & B_3(980) &= 0.02 \pm 0.07, \\ B_1(1500) &= 0.80 \pm 0.40, & B_3(1500) &= -1.32 \pm 0.14; \\ \text{Scenario II : } \bar{f}_{f_0(1500)} &= (0.49 \pm 0.05)\text{GeV}, \\ B_1(1500) &= -0.48 \pm 0.11, & B_3(1500) &= -0.37 \pm 0.20. \end{aligned} \quad (37)$$

As for the explicit form of the Gegenbauer moments for the twist-3 distribution amplitudes  $\Phi_{f_0}^S$  and  $\Phi_{f_0}^T$ , although they have been studied in the Ref. [26], we adopt the asymptotic form:

$$\Phi_{f_0}^S = \frac{1}{2\sqrt{2N_c}} \bar{f}_{f_0}, \quad \Phi_{f_0}^T = \frac{1}{2\sqrt{2N_c}} \bar{f}_{f_0}(1-2x). \quad (38)$$

The twist-2 pion distribution amplitude  $\Phi_\pi^A$ , and the twist-3 ones  $\Phi_\pi^P$  and  $\Phi_\pi^T$  have been parametrized as [16]:

$$\Phi_\pi^A(x) = \frac{f_\pi}{2\sqrt{2N_c}} 6x(1-x) [1 + 0.17(5(1-2x)^2 - 1) - 0.028(1 - 14(1-2x)^2 + 21(1-2x)^4)], \quad (39)$$

$$\Phi_\pi^P(x) = \frac{f_\pi}{2\sqrt{2N_c}} [1 + 0.21(3(1-2x)^2 - 1) - 0.11/8(3 - 30(1-2x)^2 + 35(1-2x)^4)], \quad (40)$$

$$\Phi_\pi^T(x) = \frac{f_\pi}{2\sqrt{2N_c}} (1-2x) [1 + 0.56(1 - 10x + 10x^2)]. \quad (41)$$

The  $B_s$  meson's wave function can be written as:

$$\phi_{B_s}(x, b) = N_{B_s} x^2 (1-x)^2 \exp\left[-\frac{M_{B_s}^2 x^2}{2\omega_{b_s}^2} - \frac{1}{2}(\omega_{b_s} b)^2\right], \quad (42)$$

where  $\omega_{b_s}$  is a free parameter and we take  $\omega_{b_s} = 0.5 \pm 0.05 \text{ GeV}$  in numerical calculations, and  $N_{B_s} = 63.67$  is the normalization factor for  $\omega_{b_s} = 0.5$ .

For the numerical calculation, we list the other input parameters in Table I.

TABLE I: Input parameters used in the numerical calculation[5, 27].

Masses	$m_{f_0(980)} = 0.98 \text{ GeV}, \quad m_0^\pi = 1.3 \text{ GeV},$ $m_{f_0(1500)} = 1.5 \text{ GeV} \quad M_{B_s} = 5.37 \text{ GeV},$
Decay constants	$f_{B_s} = 0.23 \text{ GeV}, \quad f_\pi = 0.13 \text{ GeV},$
Lifetimes	$\tau_{B_s^0} = 1.466 \times 10^{-12} \text{ s},$
$CKM$	$V_{tb} = 1.0, \quad V_{ts} = 0.387,$ $V_{us} = 0.2255, \quad V_{ub} = 0.00393e^{-i60^\circ}.$

 TABLE II: Decay amplitudes for  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0, f_0(1500)\pi^0$  ( $\times 10^{-2} \text{ GeV}^3$ ).

$\bar{s}s$	$F_e^{\pi,T}$	$F_e^\pi$	$M_e^{\pi,T}$	$M_e^\pi$
$f_0(980)\pi^0$ (SI)	8.98	1.50	$-3.51 + 5.75i$	$-0.01 + 0.04i$
$f_0(1500)\pi^0$ (SI)	-11.9	-1.03	$-5.89 + 4.70i$	$-0.02 + 0.04i$
$f_0(1500)\pi^0$ (SII)	17.8	2.38	$-0.77 + 3.64i$	$0.001 + 0.02i$
$\bar{n}n$	$M_a^{f_0,T} + M_a^{\pi,T}$	$M_a^{f_0} + M_a^\pi$	$F_a^{f_0,T} + F_a^{\pi,T}$	$F_a^{f_0} + F_a^\pi$
$f_0(980)(\bar{n}n)\pi^0$ (SI)	$6.3 + 6.9i$	$-0.05 + 0.03i$	$0.89 - 0.11i$	$-0.001 + 0.025i$
$f_0(1500)(\bar{n}n)\pi^0$ (SI)	$-13.8 + 20.8i$	$0.11 + 0.03i$	$-0.015 + 1.34i$	$-0.02 - 0.017i$
$f_0(1500)(\bar{n}n)\pi^0$ (SII)	$15.3 - 3.1i$	$0.01 - 0.01i$	$1.62 - 0.63i$	$0.04 + 0.04i$

If  $f_0(980)$  and  $f_0(1500)$  are purely composed of  $n\bar{n}(s\bar{s})$ , the branching ratios of  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0, f_0(1500)\pi^0$  are:

$$\mathcal{B}(\bar{B}_s^0 \rightarrow f_0(980)(n\bar{n})\pi^0) = (0.46_{-0.5}^{+0.7+1.2+0.1}_{-1.0-0.0}) \times 10^{-8}, \text{ Scenario I,} \quad (43)$$

$$\mathcal{B}(\bar{B}_s^0 \rightarrow f_0(1500)(n\bar{n})\pi^0) = (2.4_{-0.1}^{+0.1+3.4+0.5}_{-2.0-0.3}) \times 10^{-8}, \text{ Scenario I,} \quad (44)$$

$$\mathcal{B}(\bar{B}_s^0 \rightarrow f_0(1500)(n\bar{n})\pi^0) = (1.1_{-0.7}^{+0.8+1.4+0.2}_{-1.3-0.1}) \times 10^{-8}, \text{ Scenario II;} \quad (45)$$

$$\mathcal{B}(\bar{B}_s^0 \rightarrow f_0(980)(s\bar{s})\pi^0) = (1.7_{-0.1}^{+0.1+0.1+0.1}_{-0.2-0.1}) \times 10^{-7}, \text{ Scenario I,} \quad (46)$$

$$\mathcal{B}(\bar{B}_s^0 \rightarrow f_0(1500)(s\bar{s})\pi^0) = (0.66_{-0.00}^{+0.00+0.30+0.06}_{-0.13-0.05}) \times 10^{-7}, \text{ Scenario I,} \quad (47)$$

$$\mathcal{B}(\bar{B}_s^0 \rightarrow f_0(1500)(s\bar{s})\pi^0) = (3.8_{-0.4}^{+0.5+0.5+0.7}_{-0.4-0.5}) \times 10^{-7}, \text{ Scenario II,} \quad (48)$$

where the uncertainties are from the decay constant of  $f_0$ , the Gegenbauer moments  $B_1$  and  $B_3$ . One can see that the values of  $\mathcal{B}(\bar{B}_s \rightarrow f_0(n\bar{n})\pi^0)$  are smaller than the corresponding values of  $\mathcal{B}(\bar{B}_s \rightarrow f_0(s\bar{s})\pi^0)$ , it is contrary to the case of  $\bar{B}_s \rightarrow f_0(980)K^0, f_0(1500)K^0$  decays [28].

In Table II, we list values of the factorizable and non-factorizable amplitudes from the emission and annihilation topology diagrams of  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0, f_0(1500)\pi^0$ .  $F_{e(a)}^\pi$  and  $M_{e(a)}^\pi$  are the  $\pi^0$  emission (annihilation) factorizable contributions and non-factorizable contributions from penguin operators respectively. Similarly,  $F_{e(a)}^{f_0}$  and  $M_{e(a)}^{f_0}$  denote as the contributions from  $f_0$  emission (annihilation) factorizable contributions and non-factorizable contributions from penguin operators respectively.  $F_{e(a)}^{\pi(f_0),T}, M_{e(a)}^{\pi(f_0),T}$  denote

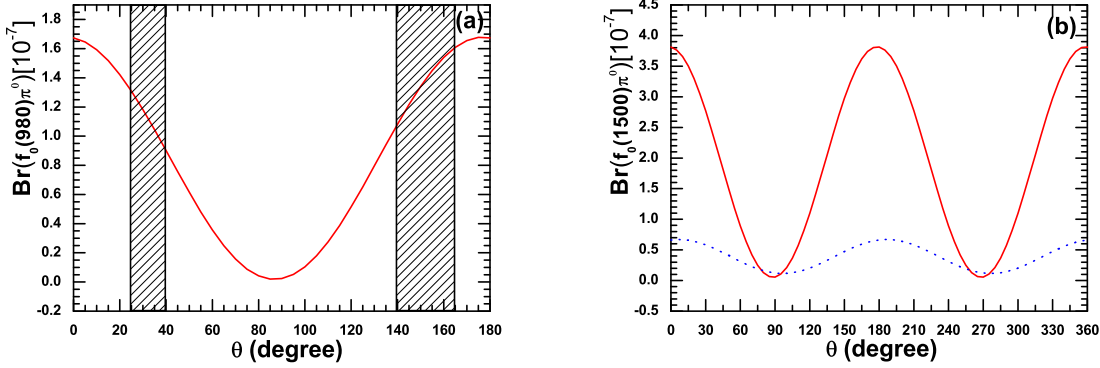


FIG. 2: The dependence of the branching ratios for  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0$  (a) and  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$  (b) on the mixing angle  $\theta$  using the inputs derived from QCD sum rules. The vertical bands show two possible ranges of  $\theta$ :  $25^\circ < \theta < 40^\circ$  and  $140^\circ < \theta < 165^\circ$ . For the right panel, the solid (dotted) curve is plotted in scenario II (I).

the corresponding contributions from tree operators  $O_2, O_1$ . From the table, regardless of the CKM suppression, one can find that the contributions from tree operators are (much) larger than the corresponding ones from penguin operators, especially for the non-factorizable emission diagrams and the annihilation diagrams. In fact, the tree operators contributions are strongly CKM-suppressed compared to penguin operators contributions. Here considering the amplitudes of annihilation diagrams is necessary.

In Fig. 2, we plot the branching ratios as functions of the mixing angle  $\theta$ . Because there are many discrepancies about the mixing of quark components for the meson  $f_0(1500)$ , we show the dependence of the branching ratio for  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$  on all the mixing angle values, that is  $(0^\circ, 360^\circ)$ . In the allowed mixing angle ranges, the branching ratio of  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0$  is:

$$1.0 \times 10^{-7} < \mathcal{B}(\bar{B}_s^0 \rightarrow f_0(980)\pi^0) < 1.6 \times 10^{-7}, \quad (49)$$

which is smaller than the branching ratio of  $\bar{B}_s^0 \rightarrow f_0(980)K^0$ . The difference is a few times even one order. As to the decay  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$ , it is interesting that this channel is better to distinguish between the first excited state (scenario I) and the lowest lying state (scenario II) for  $f_0(1500)$ . The branching ratio of  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$  for scenario I is at the order of  $10^{-8}$  in  $(0^\circ, 360^\circ)$ , while its value for scenario II is at the order of  $10^{-7}$  in most of mixing angle ranges, except for  $(60^\circ, 120^\circ)$  and  $(240^\circ, 300^\circ)$ . If the mixing angle is not in these two ranges, it is easy to determine the nature of  $f_0(1500)$ . If the observation of the decay  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$  at the order of  $10^{-7}$ , it would indicate that scenario II is favored. We also find the branching ratios of these decays are smaller than those of corresponding channels  $\bar{B}_s^0 \rightarrow f_0(980)K^0, f_0(1500)K^0$  about a few times even one order.

Now we turn to the evaluations of the CP-violating asymmetries of the considered decays in the PQCD approach. For the neutral decays  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0, f_0(1500)\pi^0$ , there are both direct  $CP$  asymmetry  $A_{CP}^{dir}$  and mixing-induced  $CP$  asymmetry  $A_{CP}^{mix}$ . The time

TABLE III: Direct  $CP$  asymmetries (in units of %) of  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0, f_0(1500)\pi^0$  decays for  $n\bar{n}$  and  $s\bar{s}$  components, respectively.

Channel	Scenario I	Scenario II
$\bar{B}_s^0 \rightarrow f_0(980)(n\bar{n})\pi^0$	51.5	-
$\bar{B}_s^0 \rightarrow f_0(980)(s\bar{s})\pi^0$	-16.0	-
$\bar{B}_s^0 \rightarrow f_0(1500)(n\bar{n})\pi^0$	29.0	15.1
$\bar{B}_s^0 \rightarrow f_0(1500)(s\bar{s})\pi^0$	19.3	-6.7

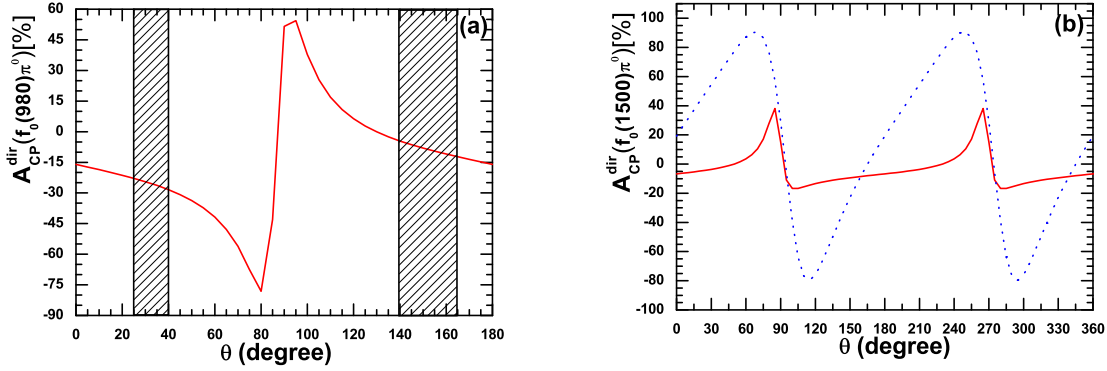


FIG. 3: The dependence of the direct  $CP$  asymmetries for  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0$  (a) and  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$  (b) on the mixing angle  $\theta$ . For the right panel, the solid (dotted) curve is plotted in scenario II (I). The vertical bands show two possible ranges of  $\theta$ :  $25^\circ < \theta < 40^\circ$  and  $140^\circ < \theta < 165^\circ$ .

dependent  $CP$  asymmetry of  $B_s$  decay into a  $CP$  eigenstate  $f$  is defined as:

$$\mathcal{A}_{CP}(t) = \mathcal{A}_{CP}^{dir}(B_s \rightarrow f) \cos(\Delta m_s) + \mathcal{A}_{CP}^{mix}(B_s \rightarrow f) \sin(\Delta m_s), \quad (50)$$

with

$$\mathcal{A}_{CP}^{dir}(B_s \rightarrow f) = \frac{|\lambda|^2 - 1}{1 + |\lambda|^2}, \quad \mathcal{A}_{CP}^{mix}(B_s \rightarrow f) = \frac{2Im\lambda}{1 + |\lambda|^2}, \quad (51)$$

$$\lambda = \eta e^{-2i\beta} \frac{\mathcal{A}(\bar{B}_s \rightarrow f)}{\mathcal{A}(B_s \rightarrow f)}, \quad (52)$$

where  $\eta = \pm 1$  depends on the  $CP$  eigenvalue of  $f$ ,  $\Delta m_s$  is the mass difference of the two neutral  $B_s$  meson eigenstates. Here we only give the direct  $CP$ -violating asymmetry.

The direct  $CP$  asymmetries of  $\bar{B}_s^0 \rightarrow f_0(n\bar{n})\pi^0, f_0(s\bar{s})\pi^0$  are listed in Table III. From the definition of the direct  $CP$  asymmetry Eq.(51) and Eq.(52), we can find the sign of

$\mathcal{A}_{CP}^{dir}$  is determined by the formula:

$$M_r^T M_i^P - M_i^T M_r^P = \begin{cases} (F_e^{\pi,T} + M_{e,r}^{\pi,T}) M_{e,i}^{\pi} - M_{e,i}^{\pi,T} (F_e^{\pi} + M_{e,r}^{\pi}), & \text{for } f_0(s\bar{s}); \\ (M_{a,r}^{f_0,T} + M_{a,r}^{\pi,T} + F_{a,r}^{f_0,T} + F_{a,r}^{\pi,T}) (M_{a,i}^{f_0} + M_{a,i}^{\pi} + F_{a,i}^{f_0} + F_{a,i}^{\pi}) - (M_{a,i}^{f_0,T} + M_{a,i}^{\pi,T} + F_{a,i}^{f_0,T} + F_{a,i}^{\pi,T}) (M_{a,r}^{f_0} + M_{a,r}^{\pi} + F_{a,r}^{f_0} + F_{a,r}^{\pi}), & \text{for } f_0(n\bar{n}). \end{cases}$$

If  $M_r^T M_i^P - M_i^T M_r^P > 0$ , the sign of the corresponding direct CP asymmetry is positive, contrarily, the value of  $\mathcal{A}_{CP}^{dir}$  is minus. So one can understand that though the penguin operators contributions are much smaller than the tree operators contributions (the difference is about two or three orders, seen in Table II), they are important to determine the direct CP asymmetry. From Table III, it is found that  $\mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(1500)(\bar{s}s)\pi^0) > 0$  in scenario I, which is contrary to the sign in scenario II. The direct reason is that there exists an opposite sign for the  $\pi^0$  emission factorizable contributions from penguin operators between these two scenarios. We also find the direct CP asymmetries for  $n\bar{n}$  components of  $f_0(980)$  and  $f_0(1500)$  are larger than those for  $s\bar{s}$  components.

In Fig.3, we plot the direct CP asymmetries of the decays  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0$  and  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$ . One can see the direct CP asymmetry of  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0$  is:

$$-30\% < \mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(980)\pi^0) < -25\%, \quad (53)$$

for the mixing angle  $\theta$  in the range of  $25^\circ < \theta < 40^\circ$ . While if the mixing angle  $\theta$  is taken in the range  $(140^\circ, 165^\circ)$ , the value of  $\mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(980)\pi^0)$  is about  $(-12 \sim -5)\%$ . For the decay  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$ , if the parameters in scenario II are used, one can find the variation range of  $\mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0)$  according to most of the mixing angles is very small, except for the values for mixing angles near  $90^\circ$  or  $270^\circ$ . while in scenario I, the variation range of  $\mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0)$  is very large. The great differences of decay constant and Gegenbauer moments of  $f_0(1500)$  result that there exists great difference for the direct CP asymmetries in two scenarios. It gives the hint that one can determine the nature of the meson  $f_0(1500)$  by comparing with the future experimental values for these direct CP-violating asymmetries.

It is noticed we consider that the meson  $f_0(1500)$  is dominated by the quarkonium content, the detail discussion can be found in [28]. If we take the mixing mechanism for  $f_0(1500)$  as  $|f_0(1500)\rangle = -0.54 |\bar{n}n\rangle + 0.84 |\bar{s}s\rangle + 0.03 |G\rangle$  [29] and neglect the small component of glueball, we have:

$$\begin{aligned} \mathcal{B}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0) &= (4.46_{-0.10-1.85-0.63}^{+0.11+2.47+0.68}) \times 10^{-8}, \quad \text{Scenario I,} \\ \mathcal{B}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0) &= (2.81_{-0.54-0.29-0.42}^{+0.61+0.32+0.47}) \times 10^{-7}, \quad \text{Scenario II;} \\ \mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0) &= -(27.5_{-0.0-0.0-3.3}^{+0.0+0.2+4.1})\%, \quad \text{Scenario I,} \\ \mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0) &= -(9.7_{-0.0-1.1-4.9}^{+0.0+1.2+5.8})\%, \quad \text{Scenario II,} \end{aligned} \quad (54)$$

which are the values corresponding to the mixing angle  $327.3^\circ$ . The uncertainties are from the decay constant of  $f_0$ , the Gegenbauer moments  $B_1$  and  $B_3$  for twist-2 LCDAs of the scalar mesons. Certainly, it is only the leading order results. In this process, the  $\pi^0$  emission factorizable amplitudes from the tree operators correspond to the color-suppressed tree amplitudes, which are known to be modified by the inclusion of the next-to-leading-order (NLO) corrections. From the calculations of the partial NLO corrections [25], our

argument is that the NLO contributions might have a small influence on the branching ratio. But it is difficult to say that the predicted discrepancy in the CP asymmetries must hold under all of the NLO corrections.

## V. CONCLUSION

In this paper, we study  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0, f_0(1500)\pi^0$  decays in the PQCD factorization approach and calculate their branching ratios and the direct CP-violating asymmetries. Several remarks are in order:

- If  $f_0(980)$  and  $f_0(1500)$  are purely composed of  $n\bar{n}$  or  $s\bar{s}$ , one can see that the values of  $\mathcal{B}(\bar{B}_s \rightarrow f_0(n\bar{n})\pi^0)$  are smaller than those of  $\mathcal{B}(\bar{B}_s \rightarrow f_0(s\bar{s})\pi^0)$ , it is contrary to the  $\bar{B}_s \rightarrow f_0(980)K^0, f_0(1500)K^0$  decays.
- In the allowed mixing angle range, the branching ratio of  $\bar{B}_s^0 \rightarrow f_0(980)\pi^0$  is:

$$1.0 \times 10^{-7} < \mathcal{B}(\bar{B}_s^0 \rightarrow f_0(980)\pi^0) < 1.6 \times 10^{-7}, \quad (55)$$

which is smaller than that of the decay  $\bar{B}_s^0 \rightarrow f_0(980)K^0$ . The difference is a few times even one order.

- The decay  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$  is better to distinguish between the lowest lying state or the first excited state for  $f_0(1500)$ . Because its branching ratios for the two scenarios have about one order difference in most of the mixing angle ranges. For example, if we take the mixing mechanism for  $f_0(1500)$  as  $|f_0(1500)\rangle = -0.54|\bar{n}n\rangle + 0.84|\bar{s}s\rangle$ , which corresponds to the mixing angle taking about  $327.3^\circ$ , one can find

$$\begin{aligned} \mathcal{B}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0) &= 4.46 \times 10^{-8}, \text{ Scenario I,} \\ \mathcal{B}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0) &= 2.81 \times 10^{-7}, \text{ Scenario II.} \end{aligned} \quad (56)$$

- There also exists great difference for the direct CP asymmetries of the decay  $\bar{B}_s^0 \rightarrow f_0(1500)\pi^0$  in two scenarios. If the parameters in scenario II are used, one can find the variation range of the value  $\mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0)$  according to most of the mixing angles is very small, except for the values corresponding to mixing angles being near  $90^\circ$  or  $270^\circ$ , while in scenario I, the variation range of  $\mathcal{A}_{CP}^{dir}(\bar{B}_s^0 \rightarrow f_0(1500)\pi^0)$  is very large. Certainly, the NLO contributions may give these direct CP asymmetries some corrections.

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